

# Unitarity bounds on scalar dark matter effective interactions at LHC

Yasuhiro Yamamoto\*

*Departamento de Física Teórica y del Cosmos, Facultad de Ciencias, Universidad de Granada,  
Granada E-18071, Spain*

\*E-mail: [yamayasu@ugr.es](mailto:yamayasu@ugr.es)

Received September 26, 2014; Revised November 11, 2014; Accepted November 18, 2014; Published December 10, 2014

.....  
We have investigated the compatibility of the unitarity bound and the 8 TeV LHC on the effective theory of scalar dark matter. In several signals of effective interactions, monojet events with missing energy were studied. We found that if the dark matter mass is about 800 GeV or heavier, the contributions of events violating unitarity are not negligible. The unitarity conditions in the 14 TeV LHC are also calculated.  
.....

Subject Index     B71, C20

## 1. Introduction

Astrophysical observations have shown that an enormous amount of dark matter exists in our universe. A promising scenario is that the dark matter is a new particle weakly interacting with the Standard Model (SM) particles. However, we have not yet found any sign of it.

Collider experiments are an important tool to shed light on it, where it could be detected as an excess of SM particles with large missing energy events. If the dark matter exists in the reach of the colliders, which is expected by the WIMP miracle, we could discover various details about it, for instance the mass, the spin, couplings with the SM, and even something about mediators. Since interactions between the dark matter and SM particles should be very weak, analyses with effective field theory have been used to obtain new information about the dark matter without assuming a certain UV structure [1,2].

ATLAS [3] and CMS [4] have submitted bounds on effective interactions between the dark matter and colored particles. They have investigated monojet events with missing energy. Dominant contributions of the process are that, after emitting a jet, collisions of two partons produce dark matter pairs. If the dark matter is heavy, much energy is required for the pair creation. In other words, much energy is injected into the effective vertex. This means that these events are simultaneously in danger of violating the unitarity bound [5,6].

If the number of events violating the bounds is not small, the given limits to the effective interactions are not reliable. This has recently been studied using several explicit simple UV completions [7–10]. However, the compatibility of these two issues had not been seriously investigated until a recent work [11] where the dark matter is assumed to be a fermion. Properties of effective interactions are different if the dark matter is a scalar field. Hence, we discuss relations between experimental bounds and the unitarity conditions of the scalar dark matter with the effective field theory.

The rest of this paper is organized as follows. In the next section, Sect. 2, we introduce the effective interactions studied in this paper. The importance of operator dimension is also pointed out. The conditions obtained are applied to collider studies, and they are compared with current experimental results in Sect. 3. The consequences of our studies are summarized in Sect. 4. The appendix is devoted to showing more general formulae for the unitarity bounds for scalar dark matter.

## 2. Effective interactions and unitarity bounds

We study the following three effective interactions,<sup>1</sup>

$$\text{the pseudo scalar interaction: } \frac{i}{M_P} \phi^\dagger \phi (\bar{q} \gamma_5 q), \quad (1)$$

$$\text{the axial vector interaction: } \frac{i}{M_A^2} \left( \phi^\dagger \overleftrightarrow{\partial}_\mu \phi \right) (\bar{q} \gamma^\mu \gamma_5 q), \quad (2)$$

$$\text{the pseudo gluon interaction: } \frac{g_s^2}{(8\pi)^2 M_{CS}^2} \phi^\dagger \phi G^{a\mu\nu} \tilde{G}_{\mu\nu}^a. \quad (3)$$

In the above interactions,  $\phi$  is the complex scalar dark matter,  $q$  stands for quarks,  $G_{\mu\nu}^a$  is the field strength tensor of the gluon, and  $g_s$  is the coupling of the SM  $SU(3)_c$ . If the operators satisfy thermal abundance, i.e.  $\langle \sigma v_{\text{rel}} \rangle \sim 0.1 \text{ pb}$ , the above suppression scales are

$$M_P \sim 130 [\text{TeV}], \quad (4)$$

$$M_A \sim 2.6 \sqrt{\frac{m_{\text{DM}}}{1 \text{ TeV}}} [\text{TeV}], \quad (5)$$

$$M_{CS} \sim 1.2 \sqrt{\frac{m_{\text{DM}}}{1 \text{ TeV}}} [\text{TeV}]. \quad (6)$$

Following the discussion in Ref. [11], these suppression scales of the above coefficients are restricted by the  $S$ -matrix unitarity of parton level  $\bar{q}q$  or  $gg \rightarrow \phi^\dagger \phi$  subprocesses,

$$M_P \geq \frac{\sqrt{s}}{8\pi} \left( 1 - \frac{4m_{\text{DM}}^2}{s} \right)^{1/4}, \quad (7)$$

$$M_A \geq \frac{1}{2} \sqrt{\frac{s}{3\sqrt{2}\pi}} \left( 1 - \frac{4m_{\text{DM}}^2}{s} \right)^{3/8}, \quad (8)$$

$$M_{CS} \geq \frac{g_s}{16\pi} \sqrt{\frac{2s}{\pi}} \left( 1 - \frac{4m_{\text{DM}}^2}{s} \right)^{1/8}. \quad (9)$$

In these expressions,  $\sqrt{s}$  is the invariant mass of the produced DM pair, and  $m_{\text{DM}}$  is the mass of the dark matter. Some of other effective interactions also appear in this order. We do not involve them in our numerical studies below since they are similar to one of our following results. Unitarity conditions including them are shown in the appendix. Differences from the real scalar dark matter are also discussed there.

<sup>1</sup> Since they are spin-dependent interactions, direct detection bounds can be evaded without tunings like the isospin violation. Several constraints to these interactions have been studied in [12]. How to obtain UV completions generating them is discussed in [7–10, 13, 14].

The qualitative consequences of the above conditions are different depending on their operator dimensions. Let us consider an effective interaction,

$$\mathcal{L} = \frac{1}{M^{D-4}} O_D, \quad (10)$$

where  $O_D$  is a dimension  $D$  operator. The cross section of the operator can be written simply as

$$\sigma \propto \frac{s^{D-5}}{M^{2(D-4)}}. \quad (11)$$

This cross section rapidly grows with increasing collision energy if the operator dimension is higher than five. Hence, the restriction by the unitarity becomes stronger if the dimensions of operators are higher. We study a dimension-five operator and two dimension-six operators. These differences of behavior are shown numerically in the next section.<sup>2</sup>

### 3. Unitarity bounds at LHC experiments

The monojet searches in the 8 TeV LHC were studied in ATLAS [3] and CMS [4]. Any excess has not yet been observed, though they have obtained lower bounds of the suppression scales. We follow the analysis of CMS because their luminosity used in the analysis is about twice that used by ATLAS.

We made a model file including the effective interactions with FeynRules [15], and generated monojet events for each interaction with MadGraph5 [16], where CTEQ parton distribution functions [17] are used. For simplicity, we have analyzed events at the parton level. Various transverse momentum cuts and the pseudo rapidity cut,  $|\eta| \leq 2.4$ , have been applied to the visible particle in each final state. In order to correct differences from the full simulation,  $pp \rightarrow Z(\rightarrow \nu\bar{\nu}) j$  has also been generated, and it is compared with the simulations of the SM background in Ref. [4]. The strongest experimental bounds to the suppression scales are obtained when the  $p_T$  cut is 450 GeV.<sup>3</sup> Estimated lower limits are shown as blue/dark gray curves in the figures below.

We checked whether each event satisfies the unitarity condition or not, and counted the number of events. Satisfying the condition does not imply the reliability of results. Events near the unitarity bound could not reproduce the same results as an effective theory even if they do not violate the unitarity. However, if events violating the condition do not occur at a low rate in a process, it is clear that given results are not reliable anymore.

In monojet events, one of colored particles in the effective vertices must be virtual, and subdiagrams with the dark matters are not  $2 \rightarrow 2$  in some of diagrams. Therefore, several events are not correctly treated in the above prescription. Their contaminations are, however, small, as discussed in Ref. [11]. We do not consider them in the following analyses.

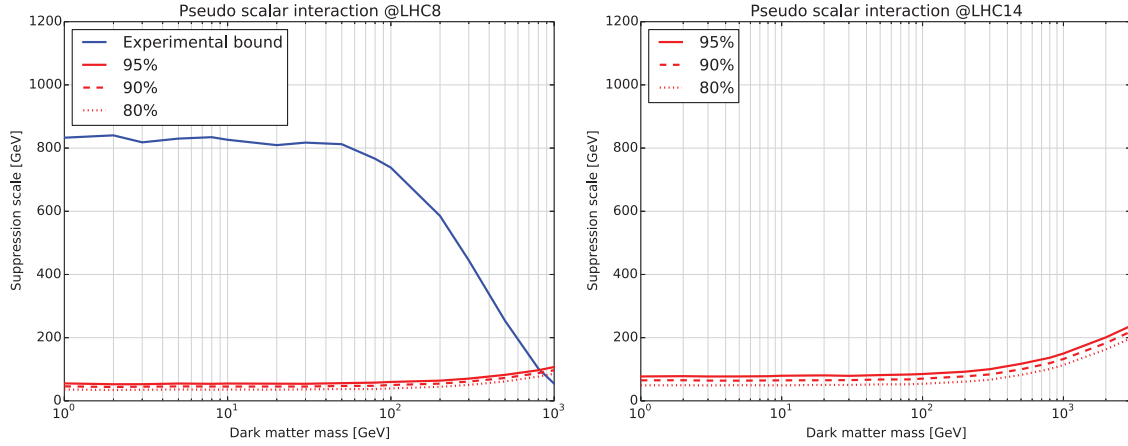
Constraints from the unitarity bounds are also studied in the 14 TeV run of LHC. Here, we have assumed that cut conditions are the same as the 8 TeV run. In a few years time, we should change these results by putting the actual bounds of the 14 TeV run if dark matter has not been detected.

#### 3.1. Pseudo scalar interaction

First, we study the pseudo scalar interaction between complex scalar dark matter and quarks. This is a dimension-five operator. The parton level cross section of this operator is almost independent of

<sup>2</sup> This feature is also observed in Ref. [11], where a dimension-seven operator has been studied.

<sup>3</sup> In this case, the parton level cross section of  $pp \rightarrow Z(\rightarrow \nu\bar{\nu}) j$  is 63.3 fb, and the given event number is 1247 with  $19.7 \text{ fb}^{-1}$ , while the full simulation result is 1460. Then, the correction factor is 1.17.



**Fig. 1.** The unitarity violation rate of the pseudo scalar interaction in LHC8 (left) and LHC14 (right). The vertical axis gives the suppression scale, while the horizontal axis shows the dark matter mass. The blue/dark gray line is the lower bound of the suppression scale calculated with experimental results in Ref. [4]. For each dark matter mass, 95%, 90%, and 80% of events satisfy the unitarity condition at the red/light gray solid, broken, and dotted lines, respectively.

collision energy, as mentioned in the previous section. Then, contributions of low-energy scatterings become relatively large because of large parton luminosity, so that the unitarity condition does not strongly restrict experimental results.

The experimental bound and the unitarity constraints are shown in Fig. 1. Because of the flat cross section, the red/light gray lines, which represent the event rates violating unitarity, almost degenerate. According to the figure, monojet analysis with this interaction is not valid if the dark matter is heavier than about 800 GeV. The bound evaluated with the experimental result of this higher-dimensional operator is not contaminated by the unitarity condition even if the given suppression scale is about 200 GeV.

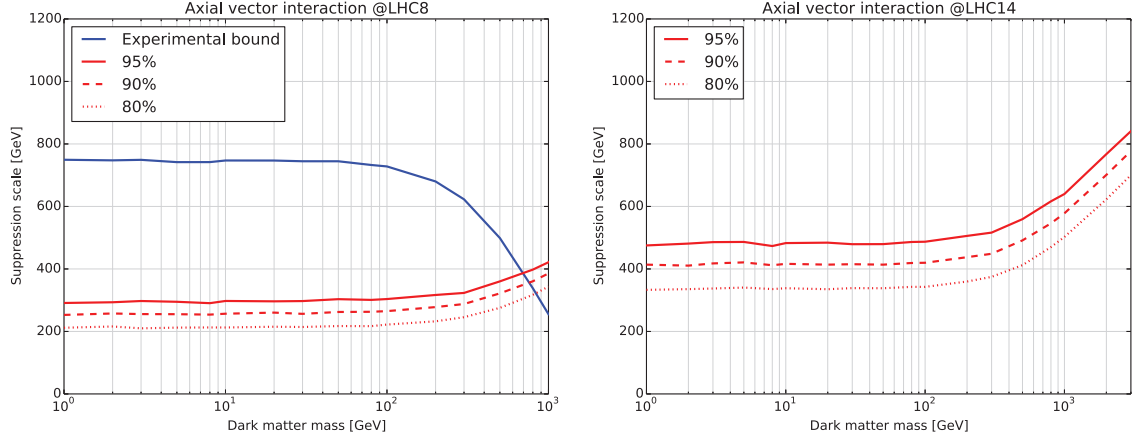
In the 14 TeV run, the condition is not greatly changed compared to the 8 TeV run because of its operator dimension.

### 3.2. Axial vector interaction

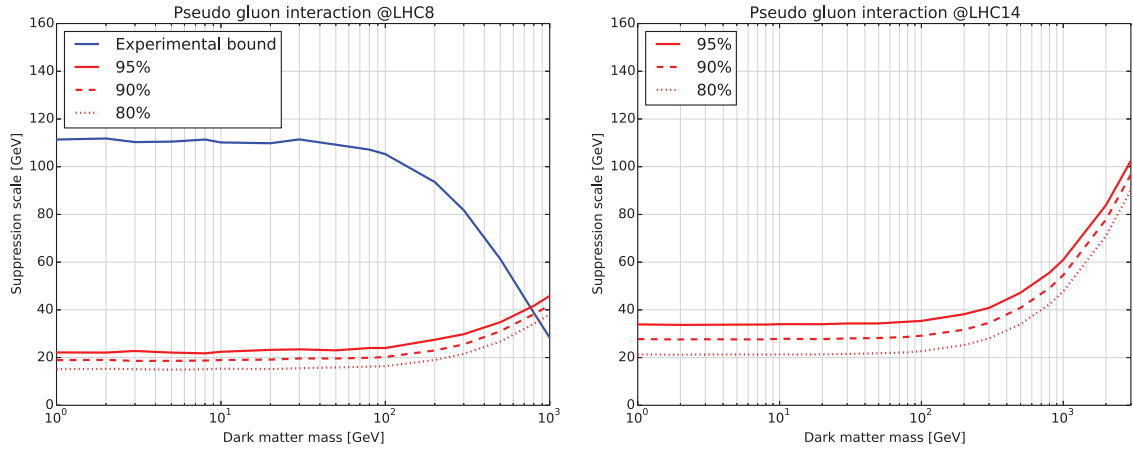
Second, we look at the axial vector interaction. The dimension of this operator is six, so that the results are qualitatively different from those above. The parton level cross section of this operator is proportional to  $s/M^4$ . To produce heavy dark matter, large collision energy is required. These events are enhanced by that factor, and simultaneously suffered by the constraint of the unitarity.

Figure 2 states 10% of events violate the unitarity if the dark matter mass is about 800 GeV. This result is numerically similar to the scalar interaction. This is because the enhancement keeps large cross sections even in the heavy dark matter region, while the unitarity conditions become stronger. Hence, the experimental bound to the scale at the point is about 400 GeV, which is much higher than that for the scalar interaction.

In the 14 TeV run, contributions of higher-energy scatterings become larger, so that the unitarity condition more strongly restricts the suppression scale. For instance, at  $m_{DM} = 1$  TeV, 10% of events violate the condition when the suppression scale is about 600 GeV in LHC14, which is 400 GeV in LHC8. The lines of the unitarity rise steeply in the TeV region.



**Fig. 2.** The unitarity violation rate of the axial vector interaction in LHC8 (left) and LHC14 (right). The meanings of the axes and lines are as for Fig. 1.



**Fig. 3.** The unitarity violation rate of the pseudo gluon interaction in LHC8(left) and LHC14(right). The meanings of the axes and lines are the same with Fig. 1.

### 3.3. Pseudo gluon interaction

Finally, the scalar dark matter which interacts with gluons is studied.<sup>4</sup> Since the gluon distribution in protons is abundant in the region of the small energy fraction, the red/light gray lines are relatively closer than those of the vector interaction; nevertheless, this operator is also dimension six—see Fig. 3.

<sup>4</sup> We have used the following formula for the running QCD coupling:

$$\frac{g_s(Q)^2}{4\pi} = \frac{\alpha_S^Z}{1 + \alpha_S^Z \frac{33-2N_f}{12\pi} \ln \frac{Q^2}{M_Z^2}}, \quad (12)$$

where  $\alpha_S^Z$  is the QCD  $\alpha_S$  at the  $Z$  boson peak and  $N_f$  is the number of the quark flavor. They are respectively 0.1184 and 5 in this paper. The scale  $Q^2$  has been chosen as the invariant mass of the dark matter pair for each event. Even if we use the geometric means of  $\sqrt{m_{DM}^2 + p_T^2}$  respecting the definition of MadGraph, the differences are numerically negligible.

For the gluon interaction, 90% of events satisfy the unitarity condition (9) if the dark matter mass is 800 GeV, which is occasionally the same with the others. At that dark matter mass, the given bound of the suppression scale is 38 GeV.<sup>5</sup>

For proton–proton collisions at 14 TeV, the number of events violating the unitarity grows rapidly in the TeV region, as for the vector interaction.

#### 4. Conclusion

Effective field theory has been used well in the dark matter search of the LHC. Since we do not know in detail the properties of the dark matter, this model-independent way is quite useful to see the strength of experimental bounds and compare the results of other experiments looking for dark matter. However, in hadron colliders, the collision energy of some events can be very large because of the compositeness of the proton. Cross sections of higher-dimensional operators become larger and larger as the collision energy increases. Therefore, the  $S$ -matrix unitarity suffers in the effective theory analyses.

In this paper, we have investigated the compatibility of the collider studies on the effective description of the interaction between complex scalar dark matter and SM colored particles. Concerning restrictions where 90% of events satisfy the unitarity, three interactions studied are not valid if the dark matter is heavier than 800 GeV in LHC8. We have also pointed out the importance of operator dimension. The 90% constraints to the suppression scales on LHC8 occasionally coincide among the three interactions. However, the unitarity condition for dimension-six operators becomes much more severe in LHC14, despite it being only a little stronger for the dimension-five operator.

The given results state that, using the effective field theory, collider searches for dark matter are not valid if its mass is heavier than several hundred GeV. For spin-independent interactions, the region is covered by direct detection experiments. On the other hand, it is found that experimental constraints are weakened there for spin-dependent interactions.

#### Acknowledgements

The author thanks M. Endo for discussions in the early stage of this work. This work has been supported in part by the Ministry of Economy and Competitiveness (MINECO), grant FPA2010-17915, and by the Junta de Andalucía, grants FQM 101 and FQM 6552.

#### Funding

Open Access funding: SCOAP<sup>3</sup>.

#### Appendix A. General formulae of the unitarity bound

We show the conditions of the  $S$ -matrix unitarity based on the more general effective Lagrangian of the complex scalar dark matter and colored particles,

$$\begin{aligned} \mathcal{L}_{\text{eff}} = & C_S \phi^\dagger \phi (\bar{q} q) + i C_P \phi^\dagger \phi (\bar{q} \gamma_5 q) + i C_V \left( \phi^\dagger \overleftrightarrow{\partial}_\mu \phi \right) (\bar{q} \gamma^\mu q) + i C_A \left( \phi^\dagger \overleftrightarrow{\partial}_\mu \phi \right) (\bar{q} \gamma^\mu \gamma_5 q) \\ & + \frac{g_s^2 C_K}{(8\pi)^2} \phi^\dagger \phi G^{a\mu\nu} G^a_{\mu\nu} + \frac{g_s^2 C_{CS}}{(8\pi)^2} \phi^\dagger \phi G^{a\mu\nu} \tilde{G}^a_{\mu\nu}, \end{aligned} \quad (\text{A1})$$

<sup>5</sup> If the operator is defined without the loop factor and the symmetric factor, the bound is about 960 GeV.

where the Wilson coefficients are expressed in the form  $C_{ij}$  instead of explicitly using the suppression scales. With the discussion in Ref. [11], the unitarity conditions are

$$\left(C_S^2 + C_P^2\right) \frac{s\sqrt{1 - 4m_{\text{DM}}^2/s}}{64\pi^2} < 1, \quad (\text{A2})$$

$$\left(C_V^2 + C_A^2\right) \frac{s^2(1 - 4m_{\text{DM}}^2/s)^{3/2}}{288\pi^2} < 1, \quad (\text{A3})$$

$$\left(C_K^2 + 4C_{CS}^2\right) \frac{g_s^4 s^2 \sqrt{1 - 4m_{\text{DM}}^2/s}}{65536\pi^6} < 1. \quad (\text{A4})$$

Since quark spins cannot be identified, conditions are averaged for initial states.

In terms of the chiral base, the conditions are written as

$$C^S C^{S*} \frac{s\sqrt{1 - 4m_{\text{DM}}^2/s}}{64\pi^2} < 1, \quad (\text{A5})$$

$$\left(C^{L2} + C^{R2}\right) \frac{s^2(1 - 4m_{\text{DM}}^2/s)^{3/2}}{576\pi^2} < 1, \quad (\text{A6})$$

where the coefficients are defined as follows:

$$C^S = C_S + iC_P, \quad (\text{A7})$$

$$C^L = C_V - C_A, \quad C^R = C_V + C_A. \quad (\text{A8})$$

The above inequalities are easily derived for the real scalar dark matter. We consider the following effective Lagrangian,

$$\mathcal{L}_{\text{eff}} = \frac{C'_S}{2} \phi'^2 (\bar{q}q) + i \frac{C'_P}{2} \phi'^2 (\bar{q}\gamma_5 q) + \frac{g_s^2 C'_K}{2(8\pi)^2} \phi'^2 G^{a\mu\nu} G^a_{\mu\nu} + \frac{g_s^2 C'_{CS}}{2(8\pi)^2} \phi'^2 G^{a\mu\nu} \tilde{G}^a_{\mu\nu}, \quad (\text{A9})$$

where  $\phi'$  is the real scalar dark matter. The perturbative unitarity conditions are

$$\left(C'^2_S + C'^2_P\right) \frac{s\sqrt{1 - 4m_{\text{DM}}^2/s}}{64\pi^2} < 2, \quad (\text{A10})$$

$$\left(C'^2_K + 4C'^2_{CS}\right) \frac{g_s^4 s^2 \sqrt{1 - 4m_{\text{DM}}^2/s}}{65536\pi^6} < 2. \quad (\text{A11})$$

Expressions with the chiral base are trivial.

## References

- [1] M. Beltran, D. Hooper, E. W. Kolb, Z. A. C. Krusberg, and T. M. P. Tait, JHEP **1009**, 037 (2010) [[arXiv:1002.4137](#) [hep-ph]] [[Search INSPIRE](#)].
- [2] J. Goodman, M. Ibe, A. Rajaraman, W. Shepherd, T. M. P. Tait, and H. B. Yu, Phys. Rev. D **82**, 116010 (2010) [[arXiv:1008.1783](#) [hep-ph]] [[Search INSPIRE](#)].
- [3] [ATLAS Collaboration], ATLAS-CONF-2012-147.
- [4] [CMS Collaboration], CMS-PAS-EXO-12-048.
- [5] M. Jacob and G. C. Wick, Ann. Phys. **7**, 404 (1959) [Ann. Phys. **281**, 774 (2000)].
- [6] I. M. Shoemaker and L. Vecchi, Phys. Rev. D **86**, 015023 (2012) [[arXiv:1112.5457](#) [hep-ph]] [[Search INSPIRE](#)].
- [7] G. Busoni, A. De Simone, E. Morgante, and A. Riotto, Phys. Lett. B **728**, 412 (2014) [[arXiv:1307.2253](#) [hep-ph]] [[Search INSPIRE](#)].

- [8] G. Busoni, A. De Simone, J. Gramling, E. Morgante, and A. Riotto, JCAP **1406**, 060 (2014) [[arXiv:1402.1275](#) [hep-ph]] [[Search INSPIRE](#)].
- [9] G. Busoni, A. De Simone, T. Jacques, E. Morgante, and A. Riotto, JCAP **1409**, 022 (2014) [[arXiv:1405.3101](#) [hep-ph]] [[Search INSPIRE](#)].
- [10] J. Abdallah et al., [[arXiv:1409.2893](#) [hep-ph]] [[Search INSPIRE](#)].
- [11] M. Endo and Y. Yamamoto, JHEP **1406**, 126 (2014) [[arXiv:1403.6610](#) [hep-ph]] [[Search INSPIRE](#)].
- [12] B. Bhattacharjee, D. Choudhury, K. Harigaya, S. Matsumoto, and M. M. Nojiri, JHEP **1304**, 031 (2013) [[arXiv:1212.5014](#) [hep-ph]].
- [13] S. Chang, R. Edezhath, J. Hutchinson, and M. Luty, Phys. Rev. D **89**, 015011 (2014) [[arXiv:1307.8120](#) [hep-ph]] [[Search INSPIRE](#)].
- [14] Y. Bai and J. Berger, JHEP **1311**, 171 (2013) [[arXiv:1308.0612](#) [hep-ph]] [[Search INSPIRE](#)].
- [15] A. Alloul, N. D. Christensen, C. Degrande, C. Duhr, and B. Fuks, Comput. Phys. Commun. **185**, 2250 (2014) [[arXiv:1310.1921](#) [hep-ph]] [[Search INSPIRE](#)].
- [16] J. Alwall, M. Herquet, F. Maltoni, O. Mattelaer, and T. Stelzer, JHEP **1106**, 128 (2011) [[arXiv:1106.0522](#) [hep-ph]] [[Search INSPIRE](#)].
- [17] J. Pumplin, D. R. Stump, J. Huston, H. L. Lai, P. M. Nadolsky, and W. K. Tung, JHEP **0207**, 012 (2002) [hep-ph/0201195].